Production of Short Bunches near γ_t

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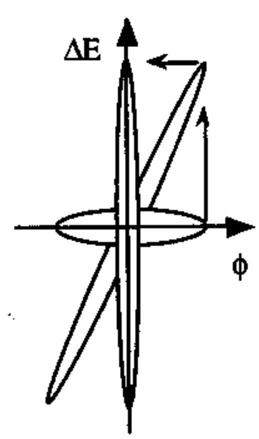
June 15, 1996

The proton driver for the $\mu\mu$ collider requires short bunches in order to minimize the longitudinal phase space entering the μ cooling system and optimize the separation of + and - polarizations. A specific requirement of 1 ns has been set, which is considerably shorter than any high current bunch produced in a high current proton synchrotron. We have examined a number of methods which should be able to reduce the bunch length.

1) Extraction near transition energy γ_t
    - Normal extraction below transition
    - Extraction using long. space charge driven oscillations $\gamma > \gamma_t$
2) RF manipulation
    - Adiabatic spreading followed by bunch rotation $\gamma_t >> \gamma$
    - Bunch rotation, γ slightly $< \gamma_t$, extraction $< \gamma_t$ or $> \gamma_t$
    - Bunch length ~ $\sqrt[4]{V_{rf}}$
    - Quadrupole modes
    - Higher frequencies / shorter buckets
3) Bunch shortening instabilities
    - Negative mass compresses above γ_t.
    - Manipulation of wall reactance
4) Coalescence of many small bunches
    - Induction linacs and external bunching ring
    - Coalescence of bunches in the accelerator
5) Chicane systems to coalesce bunches at target
6) Combine many short μ bunches in m cooling system

Of these methods the cheapest and perhaps easiest are bunch rotation using existing cavities and ssynchrotron magnets.

Of particular interest is a method which partly operates near transition where phase slip is small. Ideally, operating just at transition with the rf cavities would permit shearing of the bunch vertically in synchrotron space which would permit arbitrarily a large momentum slewing along the length of the bunch. Moving the transition gamma far above that of the beam would then permit the bunch to be rotated to the vertical position, where a short bunch would be produced. This is shown at right. The primary problem with this technique is that the dynamics of beams near transition are quite nonlinear and the nonlinearities ultimately constrain the minimum bunch length produced. This note describes the dynamics of the bunch rotation scheme and describes the limitations imposed by the



Phase Space

nonlinearities introduced by operating near transition.

The dynamics of beams near transition can be described following Cappi *et al* (IEEE Trans. on Nucl. Sci. NS-28 (81) 2389) we express the transition gamma as a function of the beam momentum $p = p_0 (1 + \delta)$, and the cirumference $L = L_0 (1 + \alpha_1 \delta + \alpha_1 \alpha_2 \delta^2 + \cdots)$, this gives

$$\gamma_t = \gamma_{t,0} [1 - (0.5 + \alpha_2) \delta] = [1 - (1.5 + w_2) \delta],$$

where

$$\alpha_1 = 1 / \gamma_{t,0}^2$$

$$\alpha_2 = -1 - 2\xi - \Delta\xi + w_2 \sim 1 + w_2$$

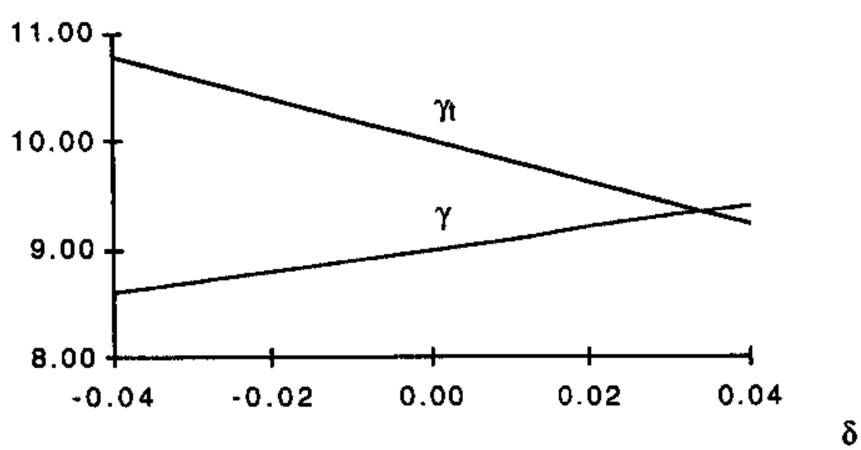
ξ = natural chromaticity ~ -1

$\Delta\xi$ = chromaticity correction ~ 0

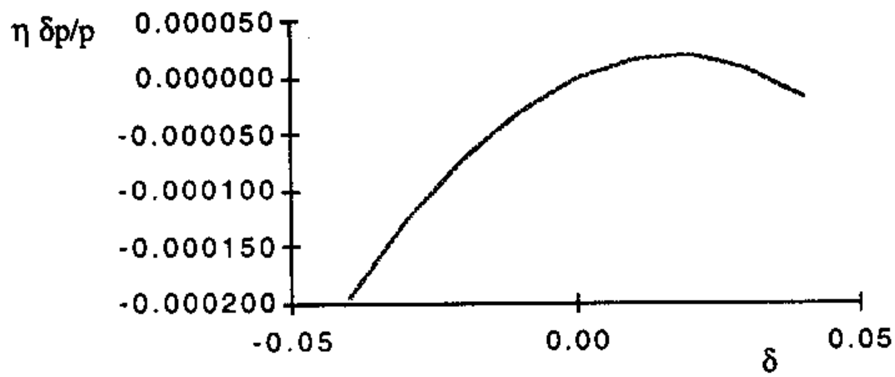
and α_{po} = Dispersion function. The machine lattice also determines

$$w_2 = \text{wiggling factor} = \int \alpha_{po}'^2 \, ds / 2 \, \alpha 1 \, L_2,$$

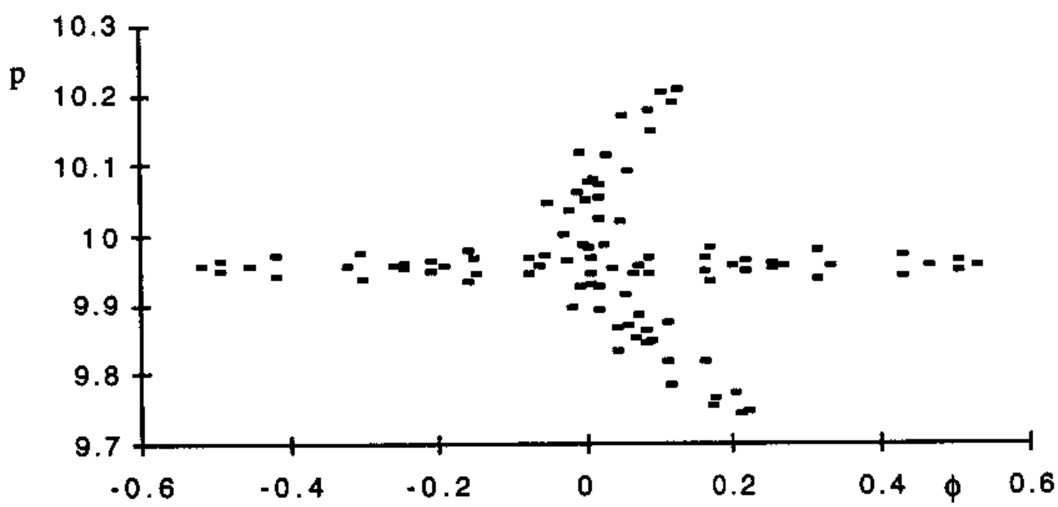
where a value of 0.44 was measured and calculated for the CERN PS. These expressions determine the longitudinal dynamics through the slip factor $\eta = 1/\gamma^2 - 1/\gamma_t^2$ and the bunching time, which should be minimized, $t = \phi_{rf} / 2 \pi f_{rf} \eta \Delta p/p$. For $\gamma = 10$ and $\Delta\gamma = 1$ these terms look like the graph below. Note that no higher order terms are included since it was experimentally determined that $\gamma_t(\delta)$ was effectively linear, with negligable δ^2 terms.



When $\Delta\gamma = \gamma_t - \gamma$ is small, the term $\eta\ \delta p/p$, which governs the horizontal slip velocity of particles in synchrotron space is essentially parabolic, since both η and δp are propotional to δ.



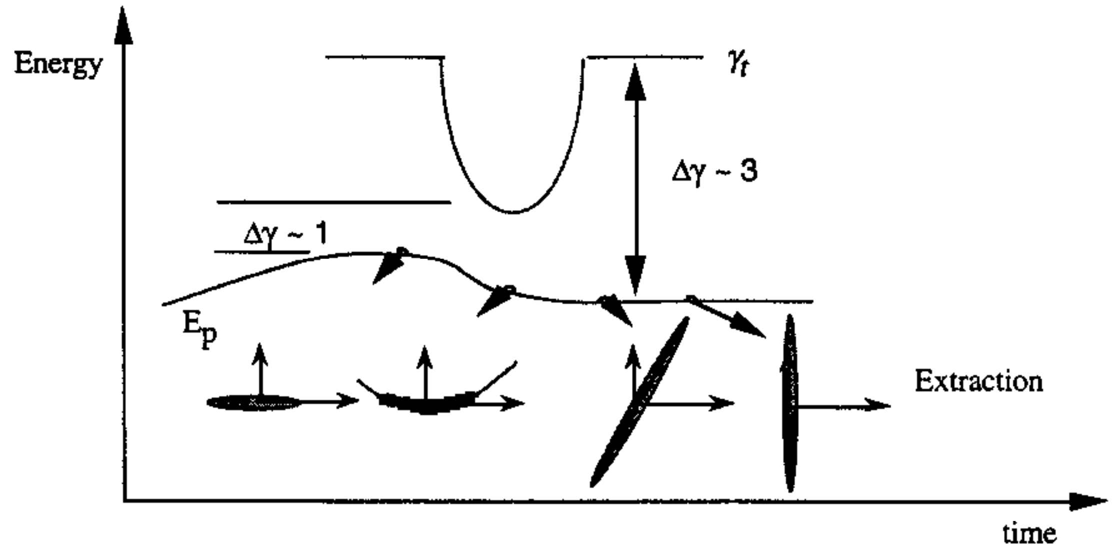
This quadratic dependance on δ will produce a quadratic dependence of the bunch shape which should prohibit small bunch lengths. This can be shown by tracking a bunch during rotation near transition. The calculation assumes that $\Delta\gamma = 1$ during the initial momentum slewing with an rf voltage of 1 MV/turn, and $\Delta\gamma \sim 4$ during the horizontal sheer with the voltage turned off. The results are shown below, plotting the 1 and 2 σ boundaries.



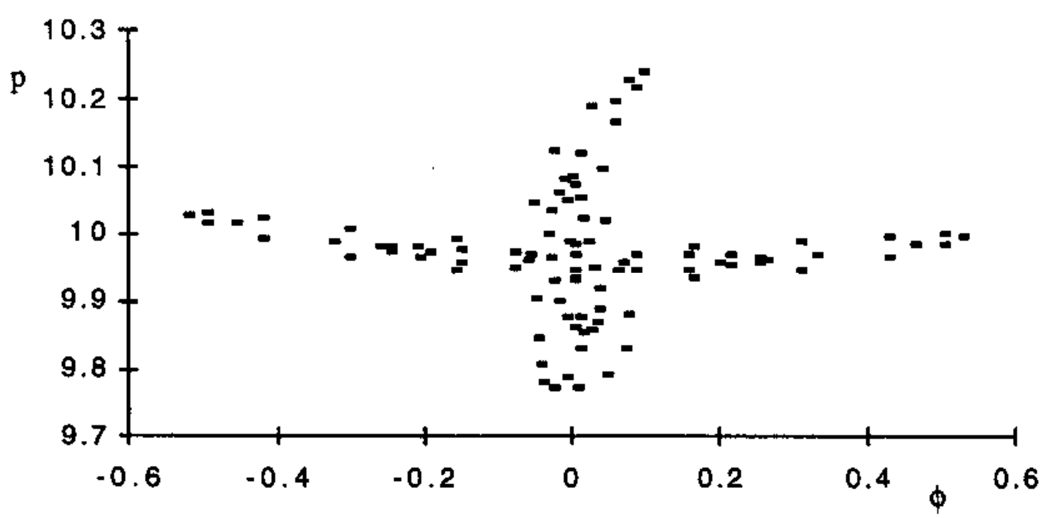
The quadratic dependence of the phase slip can be compensated by introducing a phase dependent momentum correction in the initial state. Using the expansion of the sinusoidal rf waveform

$$V\cos\theta = V(1 - \theta^2/2 + \theta^4/4 - \cdots)$$

since $1\text{-}\cos\theta \sim \theta^2$, for small θ, the nonlinearity of the rf wave itself can be used to compensate the nonlinearities near transition. This can be done by decelerating the bunch on the crest of the rf wave, with the phase spread of the beam (± 30°) providing sufficient momentum dependence for the required correction for a 4 MHz rf waveform. The whole system is shown below.



With an initial 200 MeV deceleration on the crest of the rf wave, this process produces the following results. The remaining cubic dependence of the final state on the momentum is more difficult to remove, and has been assumed to define the limits of this method.



This technique depends on the ability of the lattice to make changes in γ_t quickly and easily, which in this context means that the tunes of the machine should not change much. It seems that this can be accomplished by means of a Flexible Momentum Compaction (FMC) lattice initially proposed by Lee, Ng and Trbojevic. We have produced a lattice which might be appropriate for a 10 GeV machine based on these ideas. One period (out of 10) of this lattice is given below.

```
QF:         QUAD,    L=0.25,   K1=-.2476133
QD:         QUAD,    L=0.25,   K1=0.2642944
QF1:        QUAD,    L=0.75,   K1=-.2041294
QD1:        QUAD,    L=0.75,   K1=0.1778612
B:          SBEN,    L=4.00,   ANGLE=0.12566
B1:         SBEN,    L=1.75,   ANGLE=0.001
B2:         SBEN,    L=2.00,   ANGLE=0.06283
D1:         DRIFT,   L=0.65
D2:         DRIFT,   L=1.20
CELL:       LINE=(QF,D1,B,D1,QD,QD,D1,B,D1,QF)
CENTER:     LINE=(QF,D1,QF1,D2,QD1,D2,B1,B1,D2,QD1,D2,QF1,D1,QF)
PERIOD:     LINE=(B2,D1,CELL,CENTER,CELL,D1,B2)
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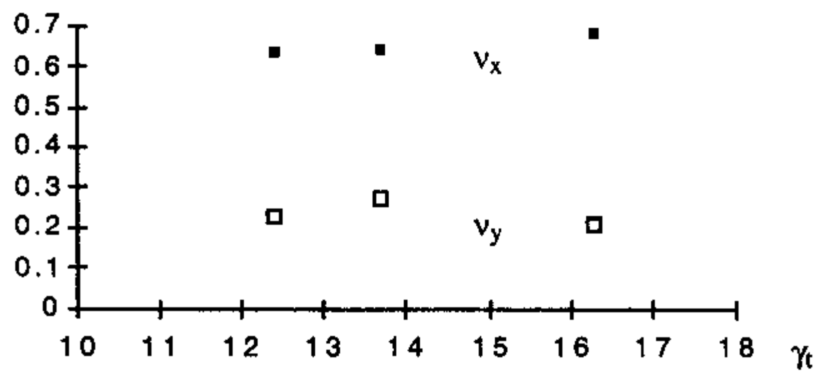
This lattice seems to have the desirable property that the γ_t is much more sensitive than other parameters to the quad settings, as shown in the plot of horizontal and vertical tunes as a function of γ_t, where the four quad settings are varied. Although the lattice was not optimized, it generates reasonable parameters.



The method outlined here seems to be fairly simple and inexpensive to implement and should have the ability to produce ~1 ns bunches from 2.5 eV-s bunches containing $1.2 \cdot 10^{13}$ protons per bunch.

A preliminary analysis of instabilities assoicated with the short bunch has shown that the bunch should be comparatively stable since: 1) the beam energy is below transition at all times, 2) the large space charge tune shift will produce considerable Landau damping, effectively limiting the coherence time to a few turns, 3) the large momentum spread will dominate the Keil-Schnell relation, raising the threshold for longitudinal microwave instability, and 4) the final bunching should take place over comparatively few turns, with the incoherent tune shift sufficiently different on each turn that resonance effects will be unlikely. We have examined the effects of structure resonances, transverse space charge, longitudinal space charge, transverse resistive wall, longitudinal microwave, high frequency cavity beam loading, Robinson instability, intra-beam scattering and charge neutralization by residual gas. These are all expected to produce significant but not necessarily pathological effects. More detail on instabilities are presented in The μ μ Collider - a Feasibility Study, May 1996.